



IMACLIM-3ME

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Contents

Contents.....	1
Introduction.....	2
1 Calibration of IMACLIM-3ME.....	4
2 Formulary of IMACLIM-3ME.....	6
2.1 Production.....	9
2.2 Households' disposable income.....	9
2.3 Final consumption and investment.....	10
2.4 International trade.....	11
2.5 Market clearings.....	12
2.6 Price system.....	13
References.....	16
Annex 1 IMACLIM-IND notations.....	17
References.....	18
Annex 1 IMACLIM-IND notations.....	19

Introduction

The IMACLIM-3ME model is a declination of the static version of the IMACLIM macroeconomic model¹ specified to approach the behaviour of the ThreeME macroeconomic model of France maintained at ADEME and OFCE.² IMACLIM-3ME performs 'comparative static' analysis at any projection horizon for any scenario explored by the Three-ME model. It is designed as the macroeconomic module of a 'macro-micro' numerical tool that combines it with a micro-simulation module of households' behaviour through the iterative exchange of variables. In the framework of this tool, IMACLIM 3-ME gradually distorts any macroeconomic equilibrium initially projected by ThreeME to reflect the iterated response of independent micro-simulation of households' behaviour.

The specifications retained after ThreeME concern both macroeconomic and microeconomic assumptions. Regarding macroeconomics, IMACLIM-3ME mirrors ThreeME by implementing a neo-Keynesian closure on imported savings under constraint (1) of investment demand proportional to the capital demand of sectors, which proceeds from trade-offs between capital and labour under valuation of capital at a price proportional to that of fixed assets. (2) Of an exogenous aggregate household trade-off between consumption and savings, dictated by microsimulation. (3) Of the assumptions of constant taxes and excise duties as well as constant public expenditures in real terms at each horizon independently of the scenario—the assumption of constant public expenditures at any given horizon reflects their indexation on (exogenous) potential growth in ThreeME. (4) And of competition on international markets regulated by an Armington specification of imperfect substitutability between domestic and imported varieties of goods for imports, and by simple price elasticities for exports.

The capital demand of sectors is calibrated on their consumptions of fixed capital (CFC) rather than on their total gross operating surpluses (GOS). The net operating surplus (NOS) is a fixed mark-up on production costs. The economy modelled in this way is demand-driven rather than supply-driven since the constraint of a fixed capital stock strictly allocated between sectors by adjustment of its rental cost is lifted. This reflects one of the main characteristics of ThreeME, with major influence on its numerical results. The constraint of a finite labour supply remains, but the labour market is not represented as a perfect market balanced by free adjustments of the average wage. Rather, a wage curve describes isoelasticity of the real wage to the unemployment rate equilibrium. In the

¹ See <http://www2.centre-cired.fr/IMACLIM>.

² See <https://www.ofce.sciences-po.fr/recherche/threeme.php>.

comparative static framework of IMACLIM-3ME, this is arguably the best possible translation of the wage-setting price-setting specification adopted by ThreeME.

Concerning microeconomics, IMACLIM-3ME only represents the substitutability of capital and labour in production, that of imports and domestic products in domestic supply, and the implicit trade-off between domestic and foreign products on international markets. In comparison to ThreeME, it thus neglects trade-offs between secondary inputs in favour of constant intensities of secondary factors, including the energy contents of productions, which remain at the levels projected by ThreeME in IMACLIM-3ME calibration data (Leontief hypothesis). It does not either represent households' consumption trade-offs, being designed to trust these to independent micro-simulation.

IMACLIM-3ME uses ThreeME elasticities to govern the trade-offs common to both models. These are the constant elasticities of substitution between capital and labour in value-added, for each sector represented. The elasticities of substitution between imported and domestic products—operating on the aggregate mass of imports of each good rather than at the level of each use as in ThreeME. And the price elasticities of the export volumes of each good. The latter elasticities operate in ThreeME around a growth trajectory of world demand addressed to France, which it is useless to reproduce in the static framework of implementation of IMACLIM-3ME (where world demand addressed to France is therefore constant at the level projected by ThreeME). A final behavioural elasticity (implicitly) calibrated on ThreeME is that of real wages to the unemployment rate, in order to ensure that any variation in the level of activity has the same feedback effect on employment in IMACLIM-3ME and ThreeME.

1 Calibration of IMACLIM-3ME

IMACLIM-3ME calibrates on datasets resulting from projections of the ThreeME model of ADEME and OFCE. The main element of each dataset is an input-output table (IOT) that details

- Inter-industrial expenses for the productions of 24 goods & services (Table 1)
- Final consumption of those 24 goods and services in 3 aggregates: households' consumption, direct public expenditure and gross fixed capital formation (immobilisation by investment).
- Value-added of those 24 productions, in 5 aggregates: net wages, social contributions on wages, consumption of fixed capital, output taxes and net operating surplus (profits)
- Imports and exports of those 24 goods and services
- Transport margins (for 4 types of transport services), trade margins, energy excise taxes, other excise taxes and value-added taxes levied on sales of those goods and services.

On top of this IOT information, ThreeME provides

- The calibration levels of a set of elements constituting the disposable income of households, pertaining to secondary income distribution. These elements are households' capital income (which aggregates corporate dividends and housing rents, real and imputed); households' perceived interests on net credit positions; households' unemployment benefits; households' other social benefits; repatriated wages; net transfers to the rest of the world; income tax payments; and other direct tax payments.
- Total population, the active population and the employed population as well as the latter's distribution across the 24 productions. The difference between the active and employed population defines the unemployed population.
- Carbon emissions attached to the uses of energy goods as inputs into the 24 production and as final consumption goods by households.
- The constant elasticities of substitution of the capital versus labour trade-offs of all productions (elasticity of 0.5 for all non-energy productions, of 0.3 for all energy productions) and of the domestic versus imported trade-offs for all aggregate

supplies ('Armington' elasticity of 0.8 for all goods); the price elasticities of exports (0.8 for all goods); and the elasticity of the average wage to the unemployment rate (-1.8).

Table 1 ThreeME / IMACLIM-3ME breakdown of goods and services

Energy goods (4)		Non- energy goods and services (20)	
COA	Coal	AFF	Agriculture, forestry and fishing
OIL	Crude and refined oil products	FPB	Manufacture of food products and beverages
GAS	Natural gas and biogas	MVH	Motor vehicles, trailers and semi-trailers
ELE	Electricity	GGP	Glass and glass products
		CBM	Ceramic products and building materials
		PAP	Articles of paper and paperboard
		ICH	Inorganic basic chemicals
		OCH	Organic basic chemicals
		PLP	Plastics products
		I&S	Basic iron and steel and of ferro-alloys
		NFM	Non-ferrous metals
		OIP	Other industrial products
		BCE	Buildings and Civil engineering
		RTR	Rail transport (Passenger and Freight)
		PTR	Passenger transport by road
		FTR	Freight transport by road and pipeline
		WTR	Water transport
		ATR	Air transport
		BSS	Business services
		PBS	Public services

Calibration on the dataset follows the standard procedure of inverting parameters and variables and solving model equations, under constraint of a set of assumptions regarding the normalisation of output prices, import prices, the valuation of fixed capital consumption, value-added and total supply per production, without loss of generality.

2 Formulary of IMACLIM-3ME

IMACLIM-3ME operates in a comparative-static framework as a system of simultaneous non-linear equations:

$$\left\{ \begin{array}{l} f_1(x_1, x_2 \dots, x_n, y_1, y_2 \dots, y_m) = 0 \\ f_2(x_1, x_2 \dots, x_n, y_1, y_2 \dots, y_m) = 0 \\ \dots \\ f_n(x_1, x_2 \dots, x_n, y_1, y_2 \dots, y_m) = 0 \end{array} \right.$$

With x_i a set of 1068 variables, y_i a set of 1403 parameters and f_i a set of 1068 functions, for some of them linear, for some of them non-linear, in x_i . The f_i functions embody constraints of either an accounting nature or a behavioural nature. The accounting constraints impose themselves on the modeller for the sake of consistency. The behavioural constraints, quite distinctively, convey the modeller's views on economic causalities and correlations.

We list and count all parameters and variables below (Table 2). The count of 1068 equations is as follows:

- Equations (4), (5), (6), (7), (8), (16), (17), (18), (22), (27), (33), (34) count 1 time: 12 equations
- Equation (28) counts 4 times (one per transport service): 4 equations
- Equation (9) counts 20 times (one per non-energy goods or services): 20 equations
- Equations (1), (2), (3), (10), (11), (12), (13), (14), (15), (19), (20), (21), (23), (24), (25), (29), (30), (31), (32) count 24 times: 456 equations
- Equation (26) count 576 times (one equation per secondary input).

Table 2 IMACLIM-3ME variables and parameters

Notation	Description	Var.	Par.
A_K	Calibration value of the cost of capital write-offs.	0	1
A_{KLi}	Calibration value of the value-added intensity of production i .	0	24
A_{iH}	Calibration value of the net interests perceived by households.	0	1
A_{Ii}	Calibration value of the final consumption of good i for investment.	0	24

A_{pU}	Calibration value of average unemployment benefits per unemployed worker.	0	1
A_{pT}	Calibration value of average other social benefits per capita in total population.	0	1
$A_{\tau Yi}$	Calibration value of the output tax τ_{Yi} .	0	24
A_u	Calibration value of the unemployment rate at power $-\sigma_{wu}$	0	1
A_{wi}	Calibration value of the average wage in the production of good i .	0	24
A_{Xi}	Calibrated parameter.	0	24
$B_{\tau Yi}$	Calibrated parameter.	0	24
C_i	Final consumption of good i by households.	20	4
G_i	Final public consumption of good i .	0	24
I_i	Final consumption of good i for the investment.	24	0
K_i	Total capital write-offs in sector i	24	0
L_i	Total labour demand from sector i	24	0
KL_i	Value-added in the production of good i .	24	0
M_i	Imports of good i .	24	0
N_T	Total population.	0	1
N_U	Unemployed population.	1	0
R_C	Consumption budget of households.	1	0
R_H	Gross disposable income of households.	1	0
S_i	Total supply of good i .	24	0
T_{CH}	Carbon tax payments by households.	1	0
W_{RoW}	Wages from non-resident firms.	0	1
X_i	Export of good i .	24	0
Y_i	Domestic output of good i .	24	0
a_{KLi}	Parameter of substitution of K to L in good i production.	0	24
b_{KLi}	Parameter of substitution of K to L in good i production.	0	24
a_{Si}	Parameter of substitution of Y to M in good i supply.	0	24
b_{Si}	Parameter of substitution of Y to M in good i supply.	0	24
e_{ji}	Carbon emissions per consumption of energy good j in production of good i .	0	96
e_i	Carbon emissions per household consumption of energy good i .	0	4
p_{Ci}	Price of good i for households.	24	0
p_{Gi}	Public price of good i .	24	0
p_{Ii}	Investment price of good i .	24	0
p_K	Cost of capital write-offs.	1	0
p_{Li}	Cost of labour input in the production of good i .	24	0
p_{Mi}	Import price of good i .	0	24

p_{Si}	Price of good i supply.	24	0
p_{Xi}	Export price of good i	24	0
p_{Yi}	Output price of good i	24	0
p_{ij}	Price of good i for the production of good j	576	0
s_{Ci}	Budget share of non-energy good i in the consumption budget of households net of energy consumptions.	0	20
t_C	Carbon tax.	0	1
t_{Ei}	Energy excise taxes (including carbon tax) per unit of consumption of good i .	0	24
t_{Ai}	Non-energy excise taxes per unit of good i consumption.	0	24
w_i	Average net wage in the production of good i .	24	0
α_{ij}	Technical coefficient, good i intensity of good j .	0	576
δ_w	Variation of wages relative to the price of the numéraire (imports) from calibration to new equilibrium.	1	0
κ_i	Technical coefficient, capital (write-off) intensity of good i .	24	0
λ_i	Technical coefficient, labour intensity of good i .	24	0
π_i	Rate of net operating surplus (mark-up rate) in the production of good i .	0	24
ρ_{KLi}	Parameter of substitution of K to L in good i production.	0	24
ρ_{Si}	Parameter of substitution of Y to M in good i supply.	0	24
ρ_T	Average <i>per capita</i> transfers benefitting to households outside unemployment benefits and pensions.	1	0
ρ_U	Average <i>per capita</i> unemployment benefits accruing to the unemployed.	1	0
σ_{KLi}	Elasticity of substitution of K to L in good i production.	0	24
σ_{Si}	Elasticity of substitution of Y to M in good i supply.	0	24
σ_{Xpi}	Elasticity of exports to the ratio of import to export prices.	0	24
σ_{wu}	Elasticity of the variation of the purchasing power of wages to the unemployment level.	0	1
τ_{CMi}	Trade margin on the sales of good i .	1	23
τ_{IT}	Income tax rate on households' gross disposable income.	0	1
τ_{LTi}	Social contribution (labour tax) rate applicable to wages in sector i .	0	24
τ_{ODT}	Ratio of direct tax payments other than the income tax to households' gross disposable income.	0	1
τ_{RoW}	Ratio of net foreign transfers by households to households' gross disposable income.	0	1
τ_S	Households' rate of savings of their gross disposable income.	0	1
τ_{TMki}	Margin of type $k \in \{RTR, FTR, WTR, ATR\}$ on the sales of good i .	4	92
τ_{VATi}	VAT rate applying to the final consumption of good i .	0	24
τ_{Yi}	Output tax rate on the production of good i .	24	0

$\omega_{\pi H}$	Share of net operating surplus (profits) accruing to households.	0	1
ω_{TH}	Share of households' carbon tax payments retroceded to households.	0	1
$\omega_{\tau Y_i}$	Share of firms' carbon tax payment variations recycled in adjustments of the output tax rate for production i .	0	24
CPI	Consumer price index.	1	0
IPI	Investment price index.	1	0
L	Total active population (labour supply) in full-time equivalents	0	1
u	Unemployment rate	1	0

2.1 Production

IMACLIM-3ME limits production trade-offs to the adjustment of capital K and labour L inputs, who combine into value-added KL following the canonical CES specification:

$KL_i = (a_{KLi} K_i^{\rho_{KLi}} + b_{KLi} L_i^{\rho_{KLi}})^{\frac{1}{\rho_{KLi}}}$. The ρ_{KLi} parameter relates to σ_{KLi} the elasticity of substitution of K to L in sector i following $\rho_{KLi} = \frac{\sigma_{KLi}-1}{\sigma_{KLi}}$. The a_{KLi} and b_{KLi} parameters are calibrated on the initial ThreeME equilibrium. They relate to the relative cost shares of K and L for the different activity sectors. Facing variations of prices p_K and p_{Li} , cost minimization of value-added yields:

$$L_i = \left(\frac{b_{KLi}}{p_{Li}}\right)^{\sigma_{KLi}} (a_{KLi}^{\sigma_{KLi}} p_K^{1-\sigma_{KLi}} + b_{KLi}^{\sigma_{KLi}} p_{Li}^{1-\sigma_{KLi}})^{\frac{-1}{\rho_{KLi}}} KL_i \quad (1)$$

$$K_i = \left(\frac{a_{KLi}}{p_K}\right)^{\sigma_{KLi}} (a_{KLi}^{\sigma_{KLi}} p_K^{1-\sigma_{KLi}} + b_{KLi}^{\sigma_{KLi}} p_{Li}^{1-\sigma_{KLi}})^{\frac{-1}{\rho_{KLi}}} KL_i \quad (2)$$

All secondary factor intensities remain constant at calibration value. They include energy intensities, which therefore remain constant at values projected by ThreeME reflecting any climate policies envisioned by the projected scenario. The KL intensities of all productions are also constant (Leontief assumption) at calibrated A_{KLi} values:

$$\frac{KL_i}{Y_i} = A_{KLi} \quad (3)$$

2.2 Households' disposable income

The disposable income of households R_H proceeds from primary factor incomes, public transfers, interests and the retroceded carbon tax, corrected from direct taxes.

$$R_H = \sum_i w_i L_i + W_{RoW} + \omega_{\pi H} \sum_i \frac{\pi_i p_{Y_i} Y_i}{1+\pi_i} + \sum_{i=U,T} \rho_i N_i + A_{iH} p_K K + \omega_{TH} T_{CH} - (\tau_{IT} + \tau_{ODT} + \tau_{RoW}) R_H \quad (4)$$

$\sum_i w_i L_i$ is the total net income from labour i.e. the sum of net wages obtained from all economic sectors. W_{RoW} is the mass of wages from foreign sources, constant at calibration value i.e. implicitly indexed on foreign prices, which collectively act as numéraire of the model (all relative foreign prices being constant at ThreeME-projected values). $\omega_{\pi H} \sum_i \frac{\pi_i p_{Y_i} Y_i}{1+\pi_i}$ is the share $\omega_{\pi H}$ of profits accruing to households in the form of real or imputed rents of housing services and dividends of other activities. Public transfers involve unemployment transfers $\rho_U N_U$ and other social transfers $\rho_T N_T$ including pensions. ρ_i stands for per capita transfers and N_i for a target population: endogenous unemployed population N_U or exogenous total population N_T . Per capita unemployment benefits are in constant (calibrated) $A_{\rho U}$ proportions to the average wage:

$$\frac{\rho_U}{w} = A_{\rho U} \quad (5)$$

Other per capita social benefits including pensions only deviate from their calibration level $A_{\rho T}$ to reflect variations of consumption prices:

$$\rho_T = CPI A_{\rho T} \quad (6)$$

The sum of net interests on net credit positions is a constant A_{iH} ratio to total capital write-offs $p_K K$ used as a proxy of total capitalisation. The retroceded carbon tax is an exogenous fraction ω_{TH} of carbon tax payments by households T_{CH} , which proceed from the exogenous carbon tax t_C and emission coefficients e_i attached to households' consumptions C_i :

$$T_{CH} = t_C \sum_i e_i C_i \quad (7)$$

Income taxes at rate τ_{IT} , other direct taxes at rate τ_{ODT} and transfers to the rest of the world at rate τ_{RoW} apply to disposable income R_H to decrease it.

2.3 Final consumption and investment

Households' saving rate τ_s is exogenous—derives from the micro-simulation to which IMACLIM-3ME is meant to couple. The consumption budget of households is equal to the disposable income net of savings:

$$R_C = (1 - \tau_S) R_H \quad (8)$$

Households' consumptions of energy goods $C_i, \forall i \in \{COA, OIL, GAS, ELE\}$ also proceed from the coupled microsimulation model (as deviations from calibration values). So do the budget-remainder shares of non-energy consumptions s_{Ci} :

$$\forall i \notin E = \{COA, OIL, GAS, ELE\} \quad p_{Ci} C_i = s_{Ci} (R_C - \sum_{j \in E} p_{Cj} C_j) \quad (9)$$

Real public expenditures G_i are constant at calibration value, to reflect ThreeME's assumption that they grow as potential output (which is the exogenous product of labour supply and labour productivity gains at all time horizons).

Mirroring the macroeconomic choices of ThreeME, investment proceeds from the capital intensities of all sectors i.e. is demand-driven. More precisely, the demands of investment goods are uniformly indexed on capital demand following calibrated A_{Ii} ratios:

$$I_i = A_{Ii} \sum_j K_j \quad (10)$$

2.4 International trade

Imported varieties of goods and services trade off with domestic varieties following Armington (1969). This means that the supply of each good or service is a CES of imports and domestic output: $S_i = (a_{Si} Y_i^{\rho_{Si}} + b_{Si} M_i^{\rho_{Si}})^{\frac{1}{\rho_{Si}}}$. The ρ_{Si} parameter relates to σ_{Si} the Armington elasticity of substitution of Y to M in sector i following $\rho_{Si} = \frac{\sigma_{Si}-1}{\sigma_{Si}}$. The a_{Si} and b_{Si} parameters are calibrated on the initial ThreeME equilibrium. They relate to the initial relative shares of $p_Y Y$ and $p_M M$ in total supply for the different activity sectors. Facing variations of prices p_{Mi} and p_{Yi} , cost minimization of supply yields:

$$M_i = \left(\frac{b_{Si}}{p_{Mi}} \right)^{\sigma_{Si}} (a_{Si}^{\sigma_{Si}} p_{Yi}^{1-\sigma_{Si}} + b_{Si}^{\sigma_{Si}} p_{Mi}^{1-\sigma_{Si}})^{\frac{-1}{\rho_{Si}}} S_i \quad (11)$$

$$Y_i = \left(\frac{a_{Si}}{p_{Yi}} \right)^{\sigma_{Si}} (a_{Si}^{\sigma_{Si}} p_{Yi}^{1-\sigma_{Si}} + b_{Si}^{\sigma_{Si}} p_{Mi}^{1-\sigma_{Si}})^{\frac{-1}{\rho_{Si}}} S_i \quad (12)$$

Exports X_i are directly elastic to relative prices:

$$X_i = A_{Xi} \left(\frac{p_{Mi}}{p_{X_i}} \right)^{\sigma_{X_i}} \quad (13)$$

with A_{Xi} a calibrated constant embodying the growth of exports projected by ThreeME. The ratio between the domestic and international vectors of prices does not result from a

direct trade balance assumption as in standard CGE models. Rather, it adjusts to allow the trade balance to hit such level that foreign savings balance out investment demand.

2.5 Market clearings

The balance of goods markets is between supply S_i , which combines domestic production Y_i and imports M_i (see international trade above), and uses, which consist of inputs into productions $\sum_j \alpha_{ij} Y_j$, households' and public consumptions C_i and G_i , immobilisations I_i and exports X_i . The public consumptions and immobilisations of energy goods are nil, by national accounting convention for the former and by definition for the latter.

$$S_i = \sum_j \alpha_{ij} Y_j + C_i + G_i + I_i + X_i \quad (14)$$

Total labour demand from sector i is:

$$L_i = \lambda_i Y_i \quad (15)$$

Labour market clearing requires that labour demand and unemployment balance out labour supply from households. One of the key structural assumptions of ThreeME and hence IMACLIM-3ME is indeed to consider equilibrium unemployment u . The labour supply of households or labour endowment L is exogenous.

$$\sum_i L_i = (1 - u) L \quad (16)$$

The unemployed population N_U is:

$$N_U = u L \quad (17)$$

A wage curve describes the correlation between the unemployment rate and the average wage (Blanchflower and Oswald, 2005). The variation of the purchasing power of wages—the variation of wages at current prices δ_w deflated by the consumer price index CPI —is elastic to unemployment u with σ_{wu} elasticity:

$$\frac{\delta_w}{CPI} = A_u u^{\sigma_{wu}} \quad (18)$$

with A_u a calibrated constant (the calibration unemployment rate at power $-\sigma_{wu}$).

Similarly to labour, total capital write-offs from sector i are:

$$K_i = \kappa_i Y_i \quad (19)$$

Capital supply is not bound but available at price p_K indexed on the price of investment goods. This reflects important neo-Keynesian features of ThreeME where close to unlimited access to credit effectively lifts any constraint on the capital stock.

2.6 Price system

Reflecting ThreeME, IMACLIM-3ME differs from the standard CGE model in terms of price and income structural constraints by accounting for non-zero profits via mark-up pricing and by considering sector-specific wages dictated by satellite labour accounts.

IMACLIM-3ME calibrates the labour inputs to productions on sectoral full-time equivalents projected by ThreeME. This induces sector-specific average net wages w_i . All w_i evolve from their calibration levels w_{i0} according to the same variation δ_w :

$$w_i = (1 + \delta_w) A_{wi} \quad (20)$$

with A_{wi} the calibration level of w_i . The endogenous adjustment of δ_w follows a 'wage curve' constraint to labour market clearing (see section 2.5). In each sector, labour costs p_{L_i} are the sum of net wages w_i and of labour taxes at constant τ_{LT_i} rates, which aggregate the social contributions of employers and employees.

$$p_{L_i} = (1 + \tau_{LT_i}) w_i \quad (21)$$

The cost of capital write-offs p_K is common to all sectors. Rather than standardly clearing capital markets, it evolves in parallel to the prices of investment goods by indexation on the investment price index IPI :

$$p_K = IPI A_K \quad (22)$$

with A_K the retained calibration value of p_K (of free choice without loss of generality). This lifts any explicit constraint on the stock of capital (see Introduction).

The producer price of good i , p_{Y_i} , proceeds from the sum of input costs, output taxes at a τ_{Y_i} rate, and a mark-up rate π_i corresponding to the net operating surplus.

$$p_{Y_i} = (\sum_j p_{ji} \alpha_{ji} + p_{L_i} \lambda_i + p_K \kappa_i + \tau_{Y_i} p_{Y_i}) (1 + \pi_i) \quad (23)$$

Input costs are the products of input prices and input intensities or technical coefficients λ_i (labour intensity), κ_i (fixed-capital-consumption intensity) and α_{ji} (intensities in secondary factors including energy intensities). The mark-up rate π_i is

calibrated on ThreeME and held constant in further runs. Depending on ThreeME scenarios, an $\omega_{\tau Yi}$ part of the variation of the carbon tax payments of sectors recycles into lower output taxes τ_{Yi} :

$$\tau_{Yi} = A_{\tau Yi} - \omega_{\tau Yi} \frac{t_C \sum_{j \in E} e_{ji} \alpha_{ji} Y_i - B_{\tau Yi}}{p_i Y_i} \quad (24)$$

with $A_{\tau Yi}$ and $B_{\tau Yi}$ the calibration values of respectively the output tax τ_{Yi} and the carbon tax payments for the production of good i : $t_C \sum_{j \in E} e_{ji} \alpha_{ji} Y_i$.

Import prices p_{Mi} remain constant at calibration values, collectively acting as numéraire of the model.

Foreign goods and services are imperfect substitutes of their domestic counterparts following Armington (1969). The trade-off between domestic and foreign goods happens at the level of total supply, a CES of imported and domestic varieties (see international trade above). Consequently, the average cost of supply derives from import prices p_{Mi} and output prices p_{Mi} following:

$$p_{Si} = \left(\alpha_{Si}^{\sigma_{Si}} p_{Yi}^{1-\sigma_{Si}} + \beta_{Si}^{\sigma_{Si}} p_{Mi}^{1-\sigma_{Si}} \right)^{\frac{1}{1-\sigma_{Si}}} \quad (25)$$

The price of intermediate goods p_{ij} , i.e. the price of good i input into the production of good j , is equal to the resource price augmented from trade margins τ_{CMi} , transport margins τ_{TM1i} to τ_{TM4i} (differentiated for four transport services), energy excise taxes t_{Ei} (which encompass any carbon tax envisaged in the ThreeME scenario on which IMACLIM-3ME calibrates) and additional non-energy excise taxes t_{Ai} :

$$p_{ij} = p_{Si} (1 + \tau_{CMi} + \sum_{k=1}^4 \tau_{TMki}) + t_{Ei} + t_{Ai} \quad (26)$$

All positive trade and transport margins remain at their calibration values. The negative margins, which correspond to those sectors providing the underlying trade and transport services, adjust to warrant the accounting balances:

$$\sum_i \tau_{CMi} p_{Si} (\sum_j \alpha_{ij} Y_j + C_i + G_i + I_i + X_i) = 0 \quad (27)$$

and

$$\forall k \in \{1,2,3,4\} \quad \sum_i \tau_{TMki} p_{Si} (\sum_j \alpha_{ij} Y_j + C_i + G_i + I_i + X_i) = 0 \quad (28)$$

The consumer prices of good i for households p_{Ci} , government p_{Gi} , investment p_{Ii} and foreign agents (export price) p_{Xi} follow definitions similar to that of intermediate prices,

with the difference of the additional value-added tax at rate τ_{VATi} for all sales but exports, which do not either bear any excise tax.

$$p_{C_i} = (p_{S_i} (1 + \tau_{CM_i} + \sum_{k=1}^4 \tau_{TMki}) + t_{E_i} + t_{A_i}) (1 + \tau_{VATi}) \quad (29)$$

$$p_{G_i} = (p_{S_i} (1 + \tau_{CM_i} + \sum_{k=1}^4 \tau_{TMki}) + t_{E_i} + t_{A_i}) (1 + \tau_{VATi}) \quad (30)$$

$$p_{I_i} = (p_{S_i} (1 + \tau_{CM_i} + \sum_{k=1}^4 \tau_{TMki}) + t_{E_i} + t_{A_i}) (1 + \tau_{VATi}) \quad (31)$$

$$p_{X_i} = p_{S_i} (1 + \tau_{CM_i} + \sum_{k=1}^4 \tau_{TMki}) \quad (32)$$

A Fisher consumer price index (CPI) measures the aggregate variation of consumer prices:

$$CPI = \sqrt{\frac{\sum_i p_{C_i} C_{i_0} \sum_i p_{C_i} C_i}{\sum_i p_{C_{i_0}} C_{i_0} \sum_i p_{C_{i_0}} C_i}} \quad (33)$$

Similarly, a Fisher investment price index (IPI) measures the aggregate variation of the prices of investment goods:

$$IPI = \sqrt{\frac{\sum_i p_{I_i} I_{i_0} \sum_i p_{I_i} I_i}{\sum_i p_{I_{i_0}} I_{i_0} \sum_i p_{I_{i_0}} I_i}} \quad (34)$$

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